A branch and bound approach for stochastic 2-machine flow shop scheduling with rework

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1 Introduction and problem statement

Turbine blades are one of the most expensive components in gas turbines for power generation, due to materials used and the complex manufacturing process. For this reason, their re-manufacturing is an economically viable approach to obtain refurbished parts for the maintenance of gas turbines. Re-manufacturing processes, differently from production of new parts, are characterized by a considerable degree of uncertainty. With respect to turbine blades, the repair process entails the removal of the hard coating and the damaged parts, the addition of the missing material through an additive manufacturing processes and their grinding. Hence, an additional material removal phase is required by means of electrical discharge technologies, to obtain the final desired shape. Within the described process, two of the most relevant re-manufacturing activities are the addition of materials through a welding process and the following grinding process. Moreover, blades quite always need to be reworked by the repetition of the same sequence of operations, thus competing for the same resources. Blades are processed in batches, consisting of a set of blades of the same stage of the turbine. The number of blades in each batch is not known in advance. In fact, some of the blades in the batch could be too damaged to be repaired and must be substituted with new ones. The processing times for each batch of blades in the different phases, included the rework ones, also entails a certain degree of uncertainty. Blades with a higher degree of damages requires longer processing times respect to less severe damages. The uncertainty associated to these factors is embedded in the processing times associated to batches of blades, described through a probability distribution.

In this paper, we focus on the scheduling of the two re-manufacturing phases described above, i.e., welding and grinding, modeling the process through a stochastic 2-machine permutation flow shop scheduling problem with rework. A set of jobs N, representing batches of blades, must be processed on two machines, M_1 and M_2 in sequence. The sequence of the jobs on the two machines is the same. After their processing on the second machine, jobs will need a rework cycle on both M_1 and M_2 (Fig 1). Rework jobs are grouped in an additional set N'. The processing time of a job $j \in N \cup N'$ on machine M_i, denoted as f_{ij}, is modeled as an independent random variable following a general distribution.

After the first processing, blades undergo an inspection to determine the parameters of the rework process. The inspection is operated offline respect to the flow shop. For this reason, in order to consider the time needed for this phase, we state that rework jobs can be processed not earlier than 2 jobs after the corresponding original job, unless the jobs to be processed are less than 2. To provide an example, let us consider a schedule referring to 4 jobs $[a, b, c, d]$ and their corresponding rework jobs $[a', b', c', d']$. Thus, a full repair schedule can only be one of the following: $[a, b, c, a', d, b', c', d']$, $[a, b, c, d, a', b', c', d']$ and $[a, b, c, a', b', d, c', d']$.

The objective function considered is the minimization of the Value-at-Risk (VaR) (Urgo, M. and Vancza, J. 2019) of the makespan, with the aim to provide a robust solution. To

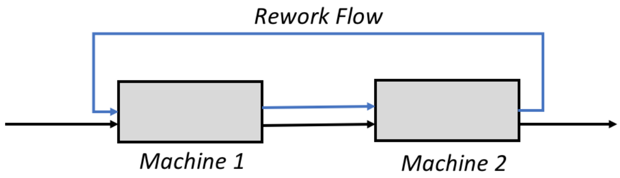

Fig. 1. Manufacturing environment

address this scheduling problem, we propose a branch and bound approach. The processing of the jobs is modeled through a Markov chain (Kulkarni, V.G. and Adlakha, V.G. 1986), whose time to absorption correspond to the makespan of the schedule, enabling the calculation of the VaR. To extend the approach beyond exponential processing times, phase-type distributions are used, due to their capability to approximate general distributions (Bladt, M. 2005).

2 Branch and bound algorithm

The branching scheme is aimed at the definition of a full schedule, containing both original and rework jobs. A forward branching scheme is used, sequencing the jobs starting from the beginning of the schedule. Due to the need to respect the constraints affecting the sequencing of rework jobs, nodes in the branching tree which are in conflict with these constraints are pruned before being evaluated.

A heuristic rule (Baker, K.R. and Trietsch, D. 2011) is exploited to obtain an initial upper bound for the search. This schedule is obtained by arranging all the jobs according to the decreasing order of $(1/E(j_1) - 1/E(j_2))$ with $E(j_1)$ and $E(j_2)$ being the expected value of the processing times of job j on machines 1 and 2 respectively. If the resulting schedule is in conflict with the constraints affecting the sequencing of rework jobs, they are shifted towards the right until the conflicts are eliminated.

To illustrate the approach for the calculation of the VaR of a schedule, let us consider an Activity on Arc (AoA) network of activities modeled as an acyclic directed graph $G = (V, A)$. Each arc in G represents an activity while the nodes in V represents states. At a given time t, an activity can only be active, dormant or idle (Kulkarni, V.G. and Adlakha, V.G. 1986). If we consider the schedule $[a, b, c, a', d, b', c', d']$, corresponding AoA network is reported in Fig.2. Hence, the set of states modeling the execution of the network, constituting the support of the Continuous Time Markov Chain (CTMC), can be obtained (Fig. 3).

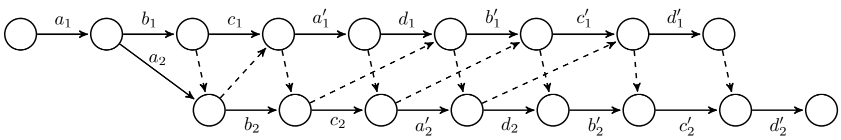

Fig. 2. AoA activity network for a two-machine flow shop with rework

Starting from this, the set of states is further enriched to consider phase-type distributions. In fact, these distribution can be in turn defined through a CTMC. Thus, each of the states in Fig. 3 represents a set of states defined by the phase type modeling the processing times of the different activities. The infinitesimal generator of the extended CTMC can be obtained, starting from the one associated to the states in Fig. 3, using a Kronecker alge-

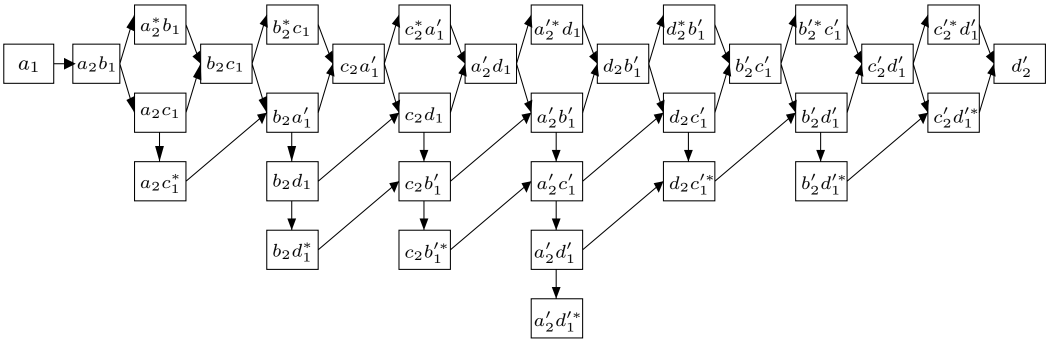

Fig. 3. States generation scheme

bra approach (Angius, A. *et. al.* 2021). The distribution of the time to absorption of the described CTMC and the quantile corresponding to the VaR_α can be obtained according to Eq. 1,

$$F(t) = 1 - \beta e^{Tt}\mathbf{1}, \quad \alpha = 1 - \beta e^{T*VaR}\mathbf{1} \tag{1}$$

The described approach applies to full schedules and can support the analysis of leaf nodes in the search tree. For the evaluation of the other nodes, let us consider a partial schedule with s jobs already sequenced. For these jobs, an approach similar to the one described for leaf nodes can be used, constituting the leftmost part of the network of activities in Fig. 2. On the contrary, for the remaining $n-s$ jobs, their processing times on the two machines can be modeled through two activities (X_1 and X_2) whose processing times are the sum of the ones of the original jobs. The AoA activity network for a partial schedule $[a, b, c, a', \ldots]$ is represented in Fig. 4. Grounding on this network, precedence constraints among operations on M_1 and M_2 for unsequenced jobs are relaxed. Since the VaR of the makespan is a regular objective function, relaxing constraints will provide a lower bound for the VaR of the complete schedules derived from the partial one considered. Thus, the approach described above can be used to generate the associated CTMC and estimate the lower bound of the VaR (Urgo, M. and Vancza, J. 2019).

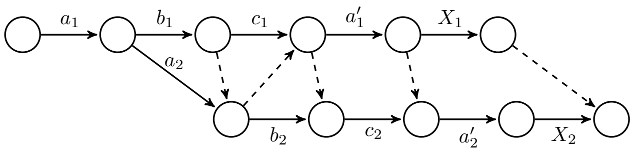

Fig. 4. AoA activity network for a partial schedule

3 Numerical experiments

A set of test instances has been generated considering $n = 6$ jobs, thus a total of 12 jobs including the rework ones. Processing times are modeled through phase-type distributions randomly generated by the BuTools library (Horvath, G. and Telek, M. 2016) by providing values for the mean and number of phases. The value of the mean is randomly sampled from three different uniform distributions with support [0,20],[30,50] and [60,80]. The number of phases is randomly sampled between 1 and 4. Different risk levels α (10 and 20%) are considered for the optimization. The results of the experiments are reported in Table 1 and Fig.5, showing the performance of the branch-and-bound algorithm in terms of solution time, number of evaluated nodes and average evaluation time per node.

Table 1. Results

Job No.	Risk level (%)	Solution time(s)				Evaluated nodes			
		Mean	Min	Max	SD	Mean	Min	Max	SD
6	10	877.6	110.8	1383.8	404.1	12262	1627	22433	6569
	20	900.9	143.6	2009.3	563.2	11442	2665	36028	9824
ALL		889.8	110.8	2009.1	480.9	11830	1627	36028	8223

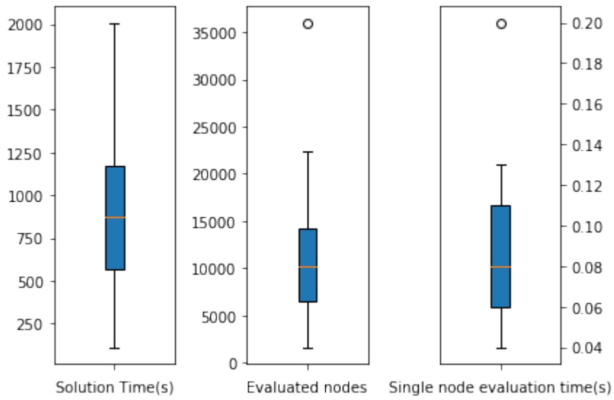

Fig. 5. Performance of algorithm

Grounding on the experiments, the proposed algorithm is able to find the optimal schedule in about 20 minutes, with 11830 nodes evaluated on average, and the average single node evaluation time is about 0.08 seconds. According to these results, the time to solve larger instance is likely to be rather large, future works will address the tighter lower bounds and effective job insertion dominance rules. Nevertheless, in the considered industrial environment, the number of jobs to schedule is in line with the one considered in the computational experiments, thus the proposed approach is valuable for the company.

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