A B&B approach for the 2-machine flow shop stochastic scheduling problem to minimize the VaR

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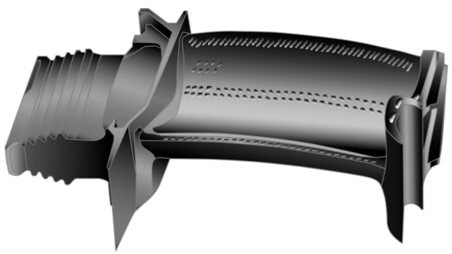
Starting from an industrial problem
Gas Turbine Blades remanufacturing process



8000 hours running

- ❑ Made of superalloys
- ❑ The whole blade as a single crystal
- ❑ Withstand the high temperatures and stresses
- ❑ One of the most expensive parts in a gas turbine

Figure 8. Comparison of unfailed and failed T-1 turbine blades used in study. Example of (a) unfailed T-1 turbine blade (b) failed T-1 turbine blade (right figure) (Zaretsky et al., 2012).

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Gas Turbine Blades remanufacturing process
Problem statement

Remanufacturing process: a blade undergoes a dissembling, thus defects are removed by means of a machining process, a reconstruction of the original shape by adding material through laser welding, and rework the blade, finally they are reassembled and sent back to the customer

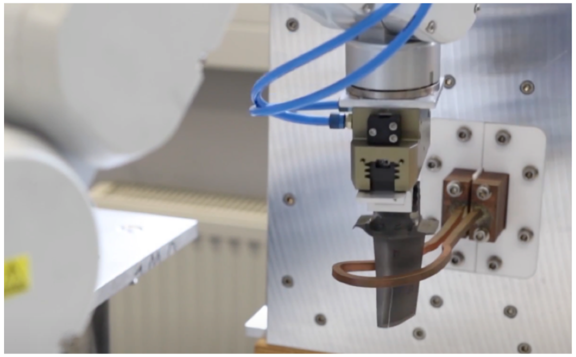
Flow Shop

The **defect removal** and **laser-based additive processes** are largely impacted by the uncertainty related to the state of the blade, the processing times could vary according to the level of damage and the machining parameters to be used

Stochastic Processing Time

The damaged turbine blades need to be finished on these **2 phases** as soon as possible

Makespan



defect removal

Laser welding

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Available Approaches

Johnson Rule

Deterministic version

Talwar Rule

- Exp distribution
- Expected Cmax

Stochastic version

B&B

- Interval pt
- Max regret

Robust version

- **limited in real manufacturing environment**
- **cannot measure stochastic feature**

- **conservative**

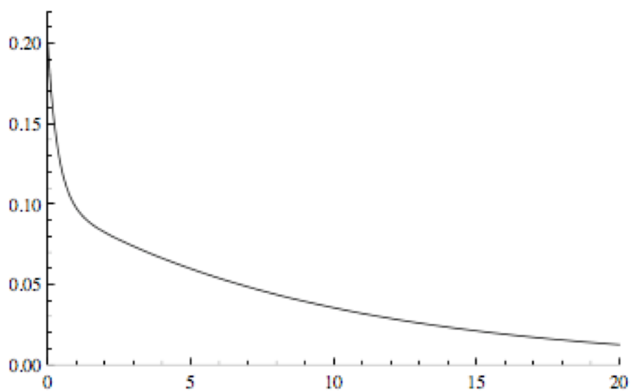
Our study: 2-machine permuted flow shop scheduling problem in which the processing time follows a general distribution and with a minimization of risk measure as objective function

Johnson, Selmer Martin. "Optimal two-and three-stage production schedules with setup times included." Naval research logistics quarterly 1.1 (1954): 61-68.
Talwar, P. P. "A note on sequencing problems with uncertain job times." Journal of the Operations Research Society of Japan 9.3-4 (1967): 93-97.
Kouvelis, Panos, Richard L. Daniels, and George Vairaktarakis. "Robust scheduling of a two-machine flow shop with uncertain processing times." Iie Transactions 32.5 (2000): 421-432.

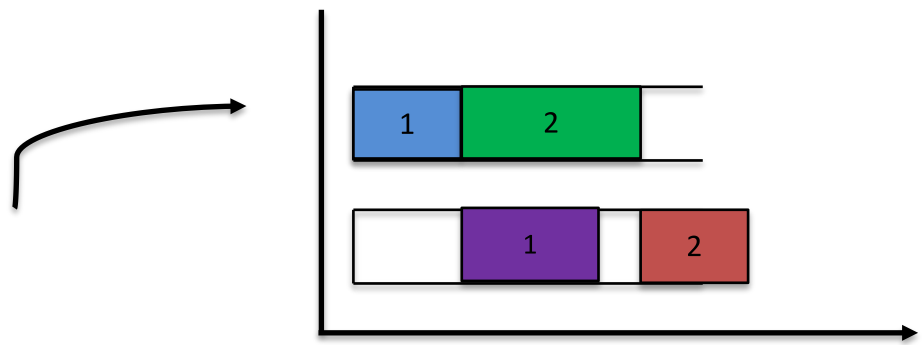
F2|perm, dis(p_{ij})|VaR[C_{max}]

2-machine permutation flow shop scheduling problem in which the processing time of each operation follows a general distribution (i.e. phase-type distribution)

Task: schedule jobs without preemption so that VaR[C_{max}] is minimal



General phase-type distribution of each operation



Why phase-type distribution

- Phase-type distributions can almost exactly approximate any distributions

- Phase-type distribution can be represented by a random variable describing the time until absorption of a Markov process with one absorbing state

- Markov Activity Networks (MAN) provide a Markov model of the execution of the jobs

- Some examples: Exponential distribution, Erlang distribution, Coxian distribution

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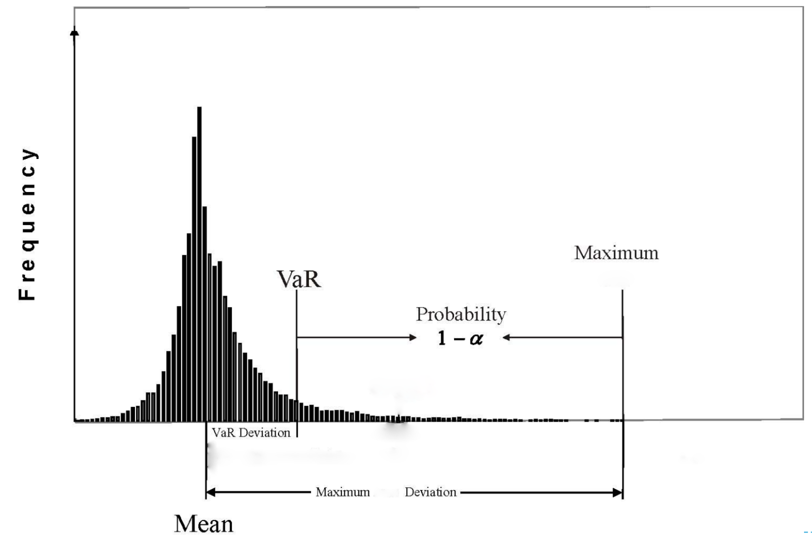
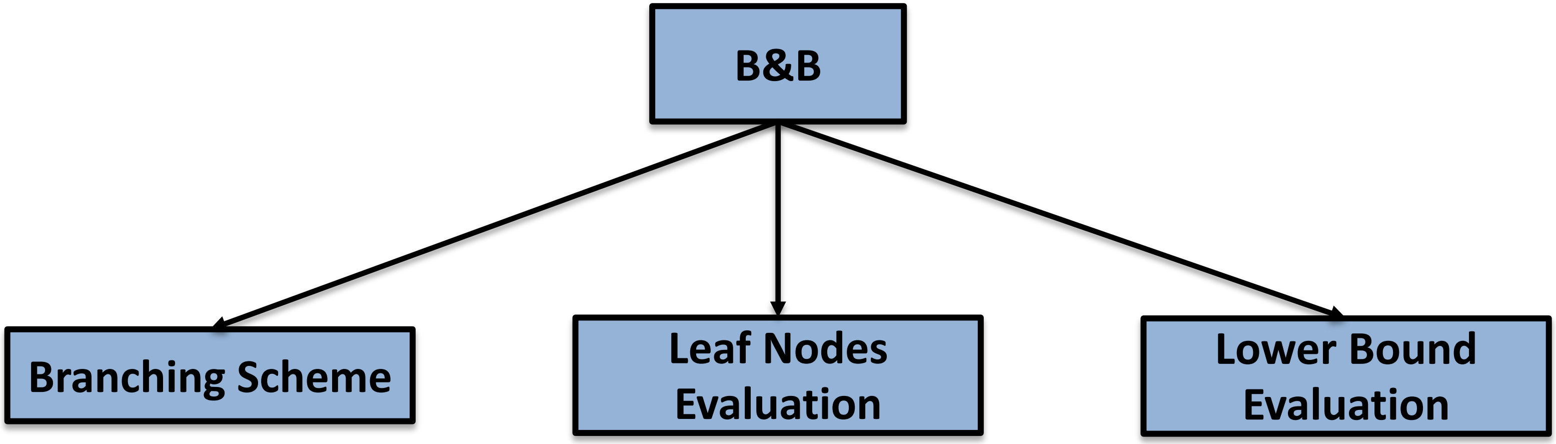
Risk-based objective function - Value at Risk

1. Describe the uncertainties associated to the processing times
2. Derive the distribution of the objective function (e.g., the makespan)
3. Evaluate the value of a measure of risk (Value at Risk)



- The maximum makespan we can experience in the best 1- α percentage of the cases
- The α level confidence that the Makespan will not excess VaR value

Approach - Branch and Bound



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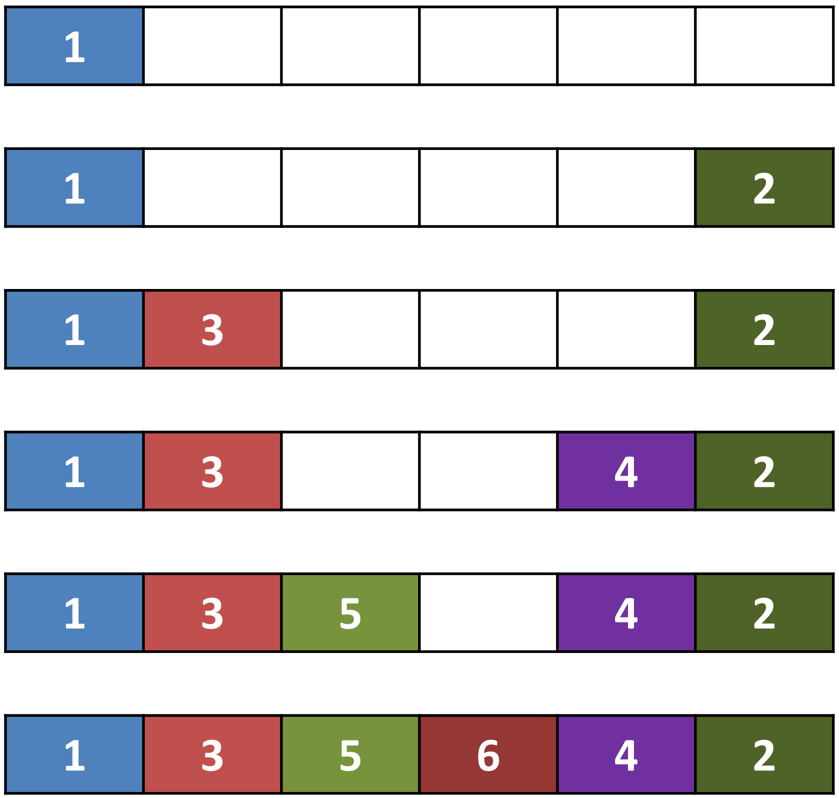
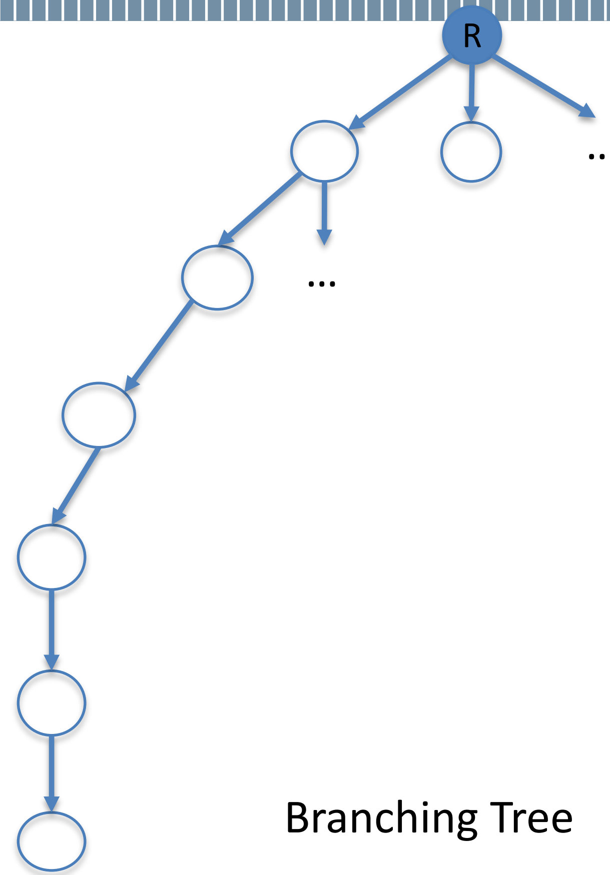
Branch and Bound - Branching Scheme





Branching Tree

The branching tree is defined by alternatively sequencing the jobs at the beginning and at the end of the schedule

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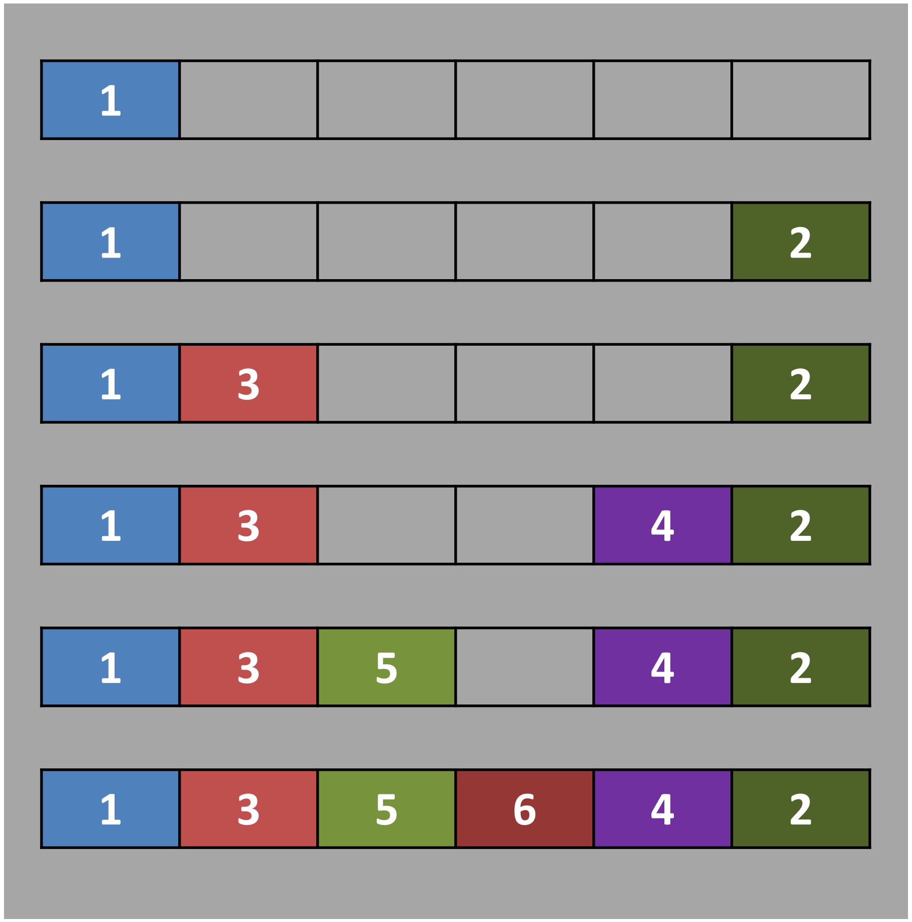
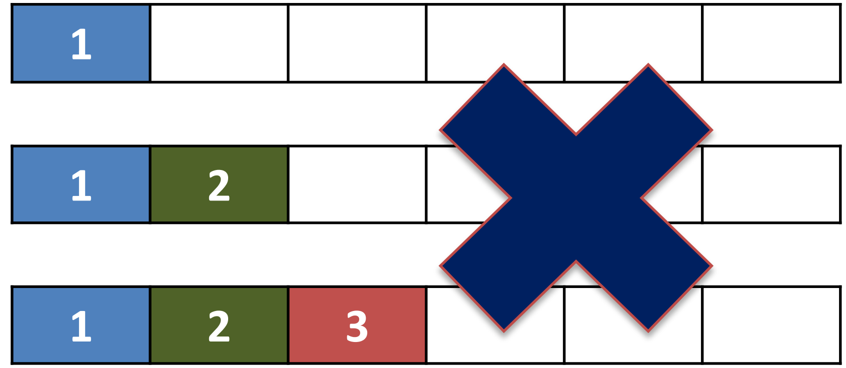
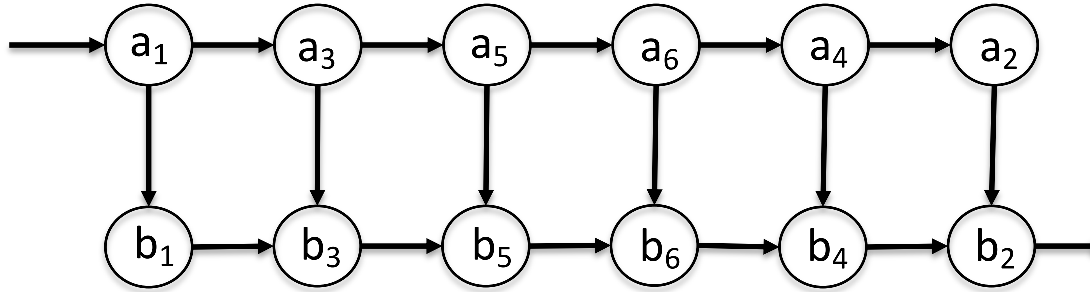
Branch and Bound - Branching Scheme
Advantages compare to a traditional branching scheme



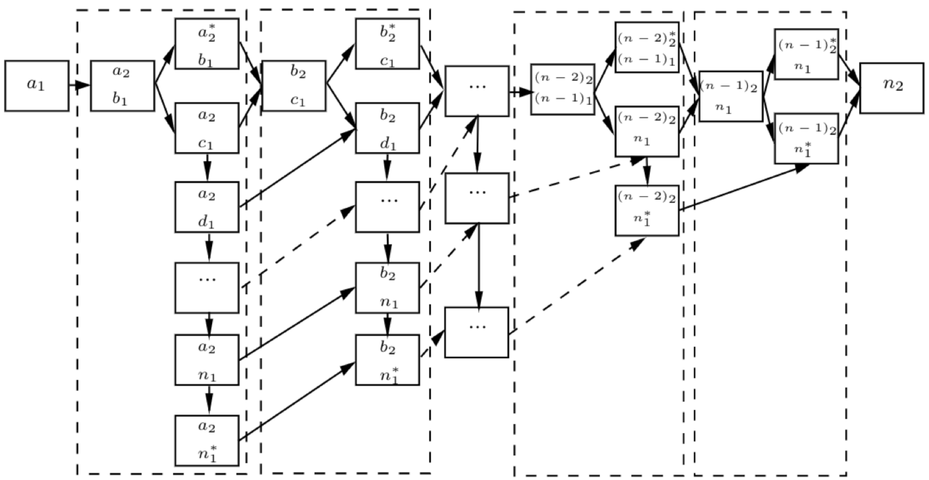


- ❑ This simple branching scheme(sequencing the jobs starting from the beginning of the schedule) can lead to very bad performance, due to the possibility of having dominance of a group of solutions
- ❑ Work for **more** instances

Branch and Bound – Leaf Nodes Evaluation



AoN (activity on nodes) of a full schedule



Markovian Activity Network

Conditions of operations in each state

- *Pending (P)* : waiting for its predecessors to complete
- *Running (R)* :being executed
- *Terminated (T)* : has been completed

Angius, Alessio, András Horváth, and Marcello Urgo. "A Kronecker Algebra Formulation for Markov Activity Networks with Phase-Type Distributions." Mathematics 9.12 (2021): 1404.

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Branch and Bound – Leaf Nodes Evaluation

Infinitesimal generator Matrix **T**

$$D(\mathbf{s}) = \bigoplus_{\forall i: s_i = R} T_i$$

i: activity is running in this state

$$O(\mathbf{s}', \mathbf{s}, i) = \bigotimes_{\forall j \in \mathcal{V}} R_j \quad \text{with} \quad R_j = \begin{cases} t_j & \text{if } j = i \\ \beta_j & \text{if } j \neq i \wedge s'_j = P \wedge s_j = R \\ I_j & \text{if } j \neq i \wedge s'_j = R \wedge s_j = R \\ 1 & \quad otherwise \end{cases}$$

activity from running to terminated

activity from pending to running

activity from running to running

$$\mathrm{F(t)} = 1 - \beta * e^{\boldsymbol{T}t}\mathbf{1}$$

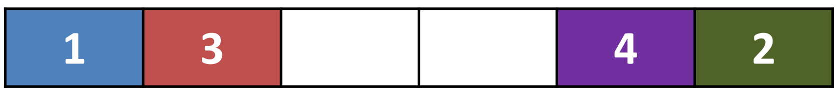
Exploit the **bisection method** to get:

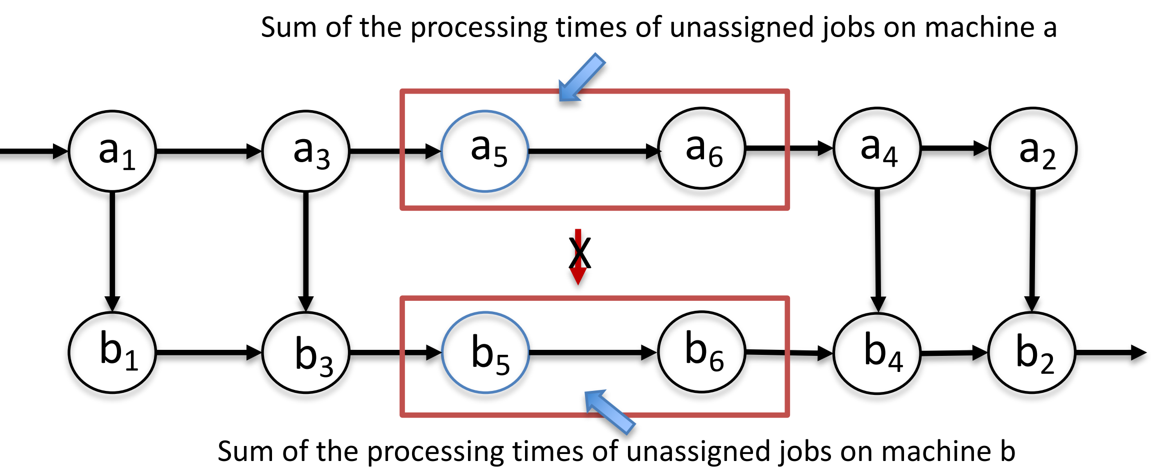
$$VaR_\alpha(\mathrm{t}) = \min\{\mathrm{z} \mid F_t(z) \geq \alpha\}$$

Angius, Alessio, András Horváth, and Marcello Urgo. "A Kronecker Algebra Formulation for Markov Activity Networks with Phase-Type Distributions." Mathematics 9.12 (2021): 1404.

Branch and Bound – Lower Bound Evaluation

In order to evaluate non-leaves nodes, we must be able to evaluate partial schedules.
We use the same approach to derive the LB of VaR of Makespan for a partial schedule.





AoN (activity on nodes) of a partial schedule

LB: Makespan is a regular objective function, relaxation of constraints will give the lower bound for this partial schedule

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Experiments

We generate 30 10-jobs instances. The processing time distribution of each operation was randomly generated by *Butools* starting from its mean and the number of phases.

Job Num.	Solution time(s)			Evaluated Nodes		
10	Avg.	Min.	Max.	Avg.	Min.	Max.
	106	17.88	249	408	217	792

10 jobs enumeration tree nodes: 9,864,100

BuTools (2018) BuTools 2.0. URL http://webspn.hit.bme.hu/ telek/tools/butools/

Conclusion & Future work

Conclusion

1. Stochastic 2-machine flow shop scheduling with general distributed processing times
2. minimization of the VaR of the makespan
3. The execution of activities is modelled as a Markovian Activity Network
4. Branch and bound algorithm to find the optimal schedule

Future work

1. Exploit heuristic approaches to find an initial solution
2. Exploit dominance rules to speed up the exploration of the branching tree
3. Test the approach on larger instances
4. Extend to different problems(release date, tardiness…)

Thanks for your attention!