Exact, heuristic, and mat-heuristic approaches for stochastic optimization problem

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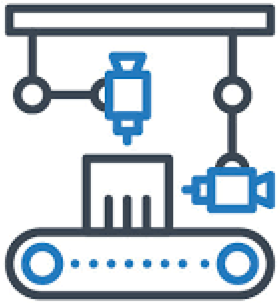
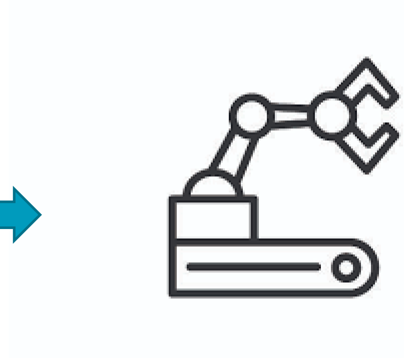
Oct 2023

Example problem

Manufacturing parts



Machine 1



Machine 2

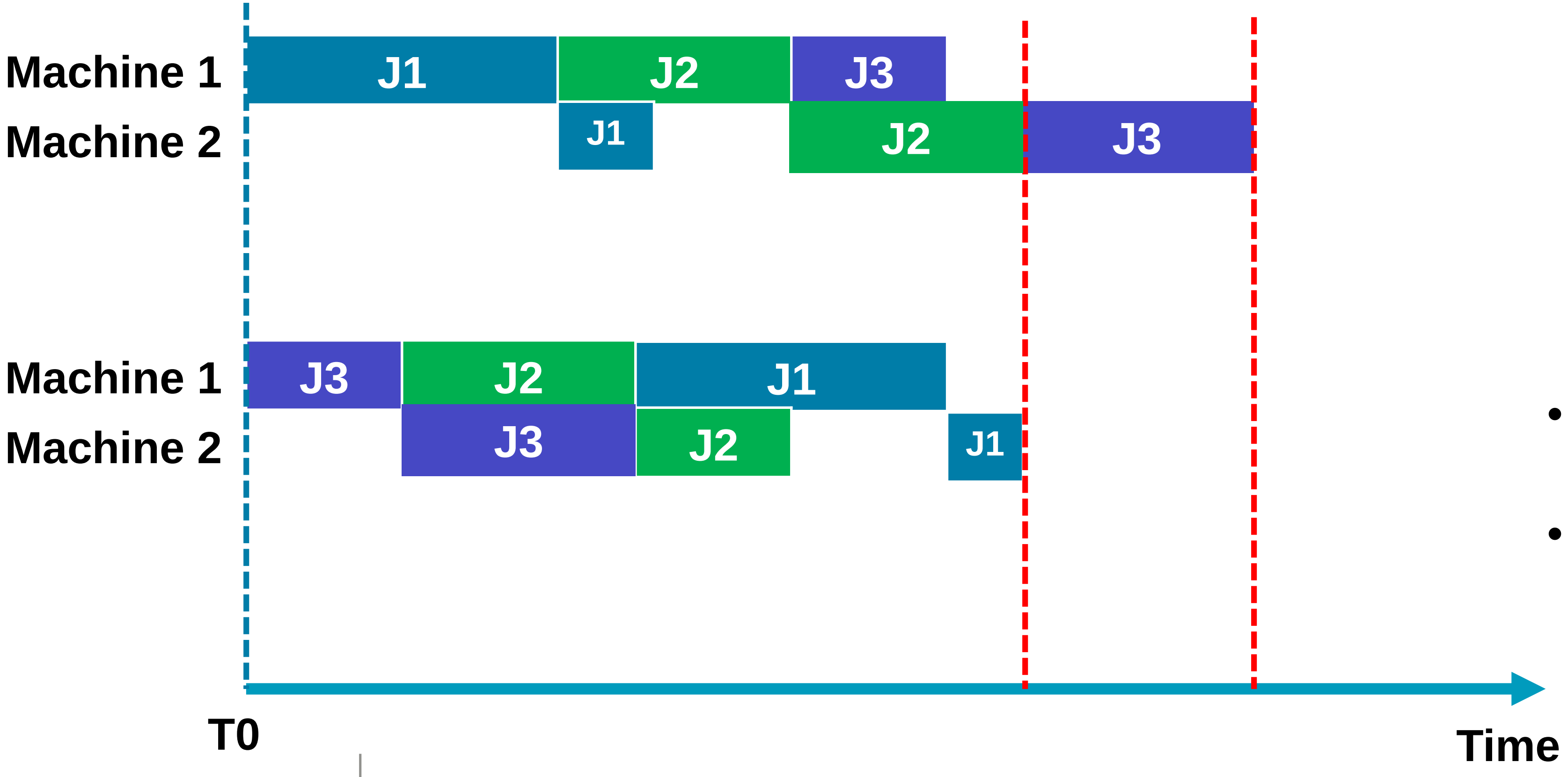
EXIT

A set of different manufacturing parts will go through two machines with **known** **different** processing times

What does scheduling do?

Scheduling: Decide the processing order of the jobs

Gantt Chart



Schedule	Time
1,2,3	13
1,3,2	12
2,1,3	12
2,3,1	10
3,1,2	12
3,2,1	10

- Find the smallest one(at least one)
- Solve up to **200-500** jobs quickly

This world is not always deterministic

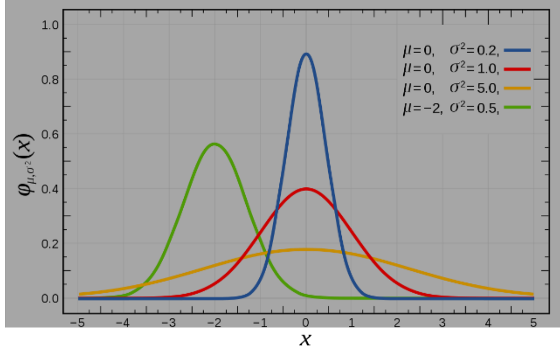
In Economics, Profit — earned by the entrepreneur who makes decisions in an uncertain environment — is the entrepreneur's reward for bearing uninsurable risk.

What if the processing time of each job on each machine is uncertain?

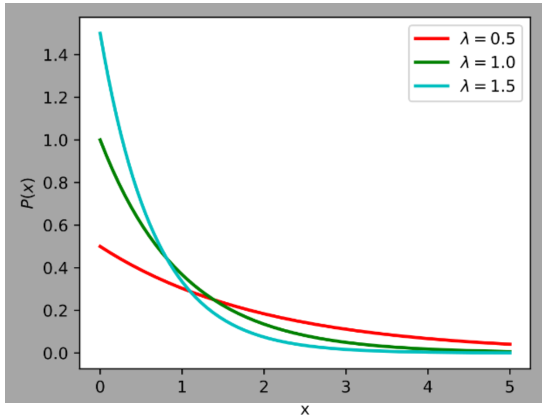
- How to make scheduling decisions?
- How to mitigate the uncertainties?

How to describe the uncertainties

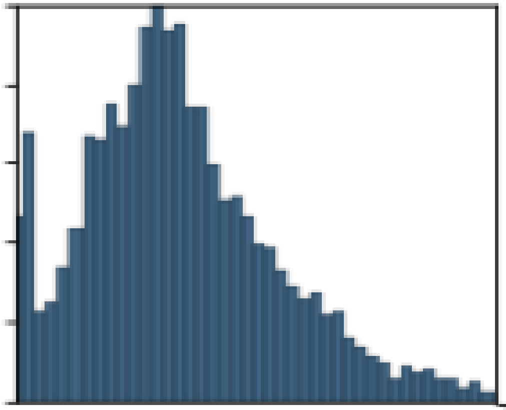
- Distribution



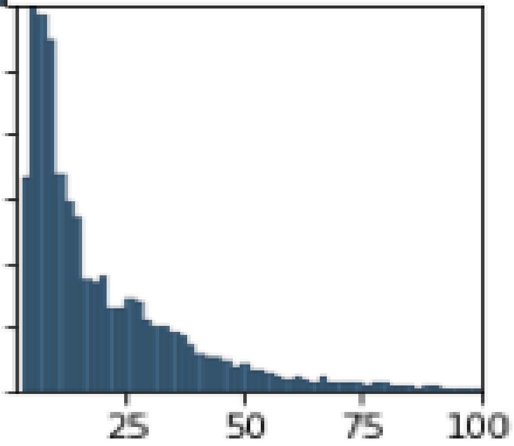
Normal distribution



Exponential distribution



Multi-mode



Heavy tail

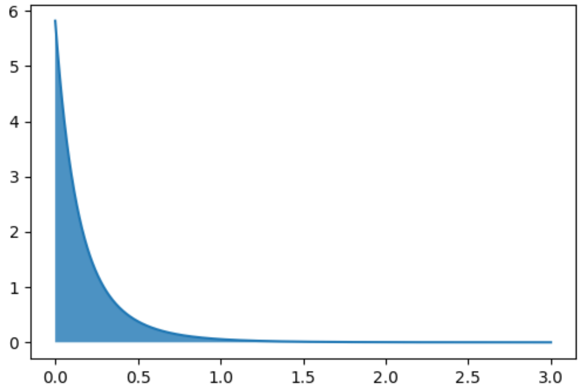
Phase-type distribution to fit general distribution

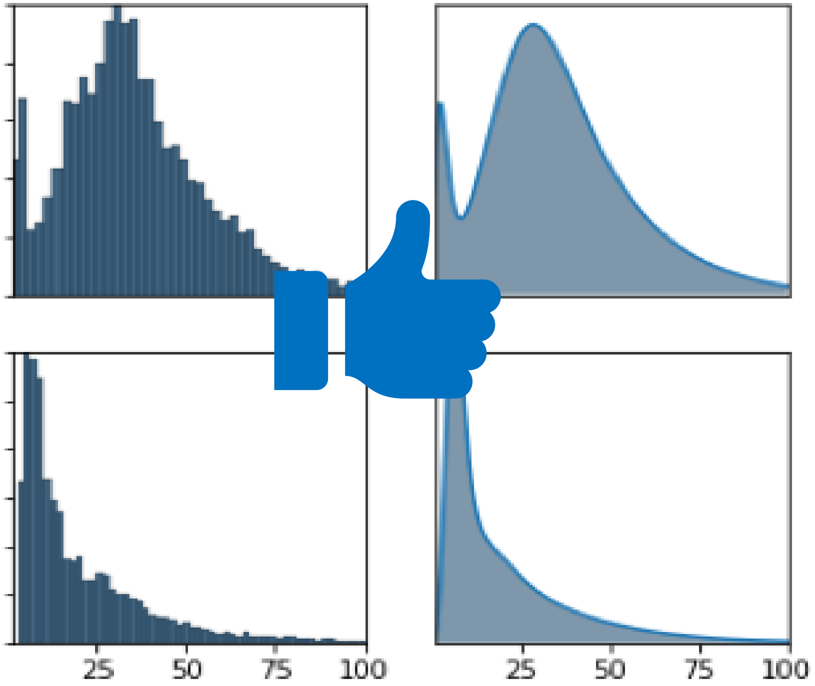
- A distribution described by **Markov chain**
- Fit **any** kind of distribution, the cost is the dimension of the matrix

$$\beta = [\,0.5 \quad 0 \quad 0.5\,]$$

$$T = \begin{bmatrix} -2.4 & 0.91 & 0.74 \\ 1.3 & -2.5 & 1.2 \\ 0 & 0.85 & -0.85 \end{bmatrix}$$

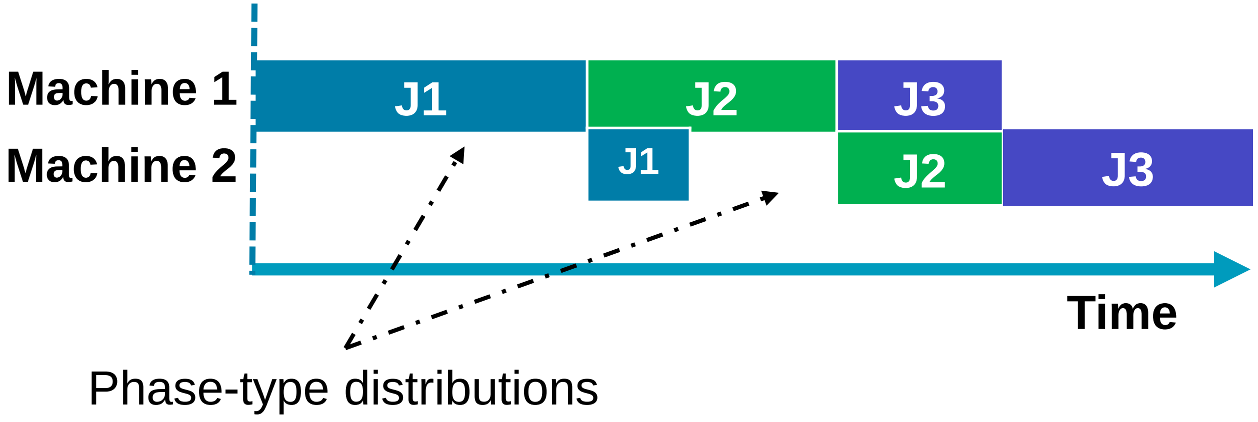
$$\Downarrow$$



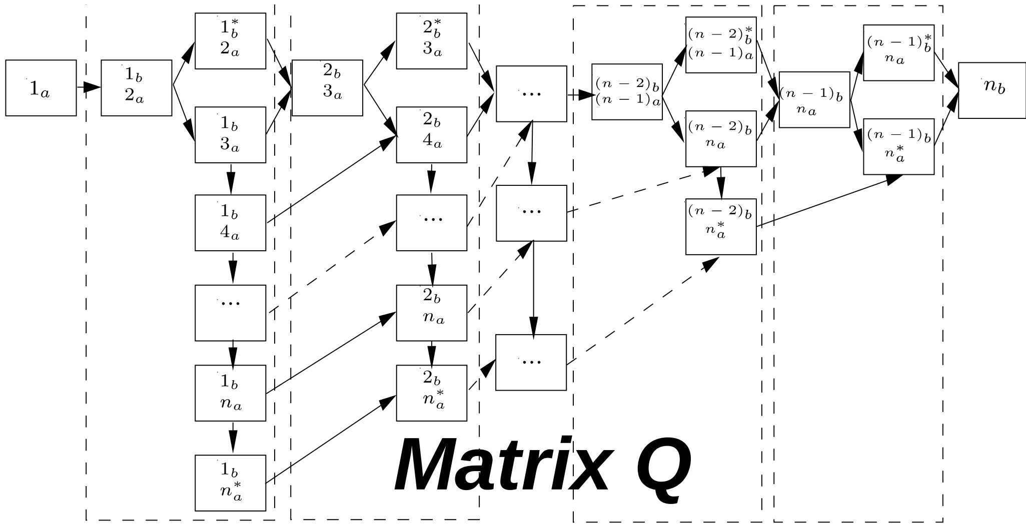


Empirical Phase-type fitted

How to calculate the completion time distribution?



Continuous time Markov Chain



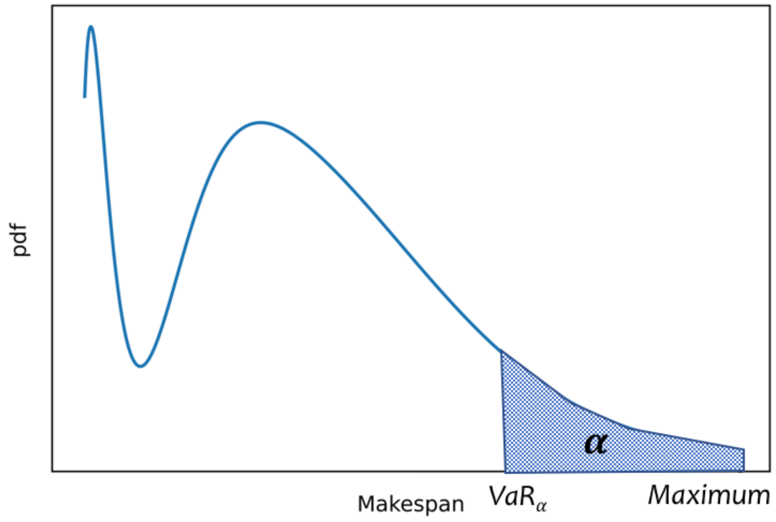
$$F(t) = 1 - \beta * e^{t*Q}\mathbf{1}$$

Distribution of Completion time

How to assess the completion time distributions?

Schedule [1,2,3], distribution of completion time, $\mathrm{F}(\mathrm{t})$

- ➢ Value-at-risk, i.e., quantile



✓ Set a risk level α based on the manager's *risk preference*

✓ We have $1-\alpha$ level confidence that the completion time will **not** excess this VaR value

Find the optimal schedule with uncertainty

Instead of comparing distributions, the decision-making becomes **single value (VaR)** comparisons under same risk level

No.	Schedule	VaR 10% of completion time
1	1,2,3	21
2	1,3,2	19
3	2,1,3	32
4	2,3,1	17
5	3,1,2	9
6	3,2,1	44

Is Brute Force always good?

# jobs	# possible schedules		~Time
3	3!	= 6	0.6 s
4	4!	=24	2 s
5	5!	=120	12 s
6	6!	=720	1 m
7	7!	=5040	8 m
8	8!	=40320	1 h
9	9!	=362880	10 h
10	10!	=3628800	100 h

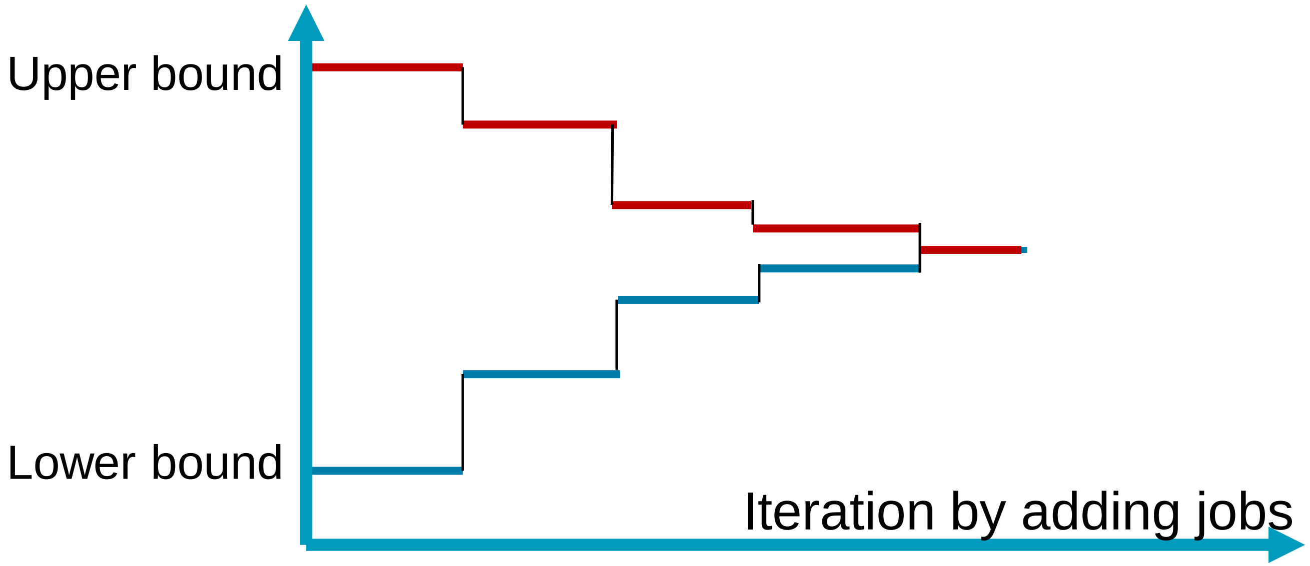
Calculation of one schedule $\approx$ 0.1s

?

How to find the optimal schedule as fast as possible

Exact branch-and-bound approach

Assign jobs one by one, calculate the lower bound and upper bound, to approach the best full order



It was implemented to solve up to **20** jobs instances in 1 hour to find the **optimal** schedule

Lei Liu, Marcello Urgo. Risk-based robust production scheduling: a branch-and-bound approach for the stochastic two-machine flow shop scheduling problem to minimise the value-at-risk, *International Journal of Production Research*

What’s after exact approach?

- 20 jobs
- Optimality (confident the result is the best schedule)
- Too small

- Slove larger problem
  # jobs >20
- Even sacrifice some optimality → sub-optimal

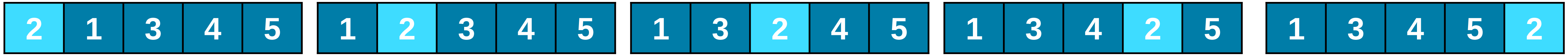
Heuristic is a technique designed for problem solving more quickly when fail to find any exact solution. This is achieved by trading optimality or accuracy for speed. It can be considered a shortcut. **- *Wikipedia***

Heuristic approach- Iterated Greedy / Local Search

1. Initial schedule

2. Random chosen one job,
   Take it out

3. Iterated insert and keep the best performed schedule (calculation with CTMC and VaR)



4. Repeat step 2- step 3, until no improvement, EXIT

Heuristic approach- Iterated Greedy / Local Search

- This heuristic can solve up to 30 - 50 jobs in 1 hour
- With relatively good performance, grounding on the improvement on the initial solution, but cannot demonstrate optimality

Lei Liu, Marcello Urgo. Robust scheduling in a two-machine re-entrant flow shop to minimise the value-at-risk of the makespan: a branch-and-bound and heuristic algorithms based on Markovian activity networks and phase-type distribution, under review at *Annals of Operations Research*.

Extension directions

- **Stochastic** scheduling problem can be extended to **various operations management problems**, e.g., *vehicle routing problem*, *facility location problem*, etc.

- Besides Value-at-Risk, i.e., quantile, ***conditional value-at-risk***, i.e., expected shortfall, is popular in finance area, which can also be borrowed.

Welcome to collaborate

Methodologies

- **Stochastic decision-making** with risk measures
- **Integer programming** and constraint programming
- **Machine learning** and artificial intelligence
- Markov analysis

Applications

- Operations management / Industrial engineering
- Machine scheduling and project management
- Online retail

Welcome to collaborate on research projects!

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